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Electron–phonon drag effect at 2D Landau levels

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Abstract. We propose a theory of the electron–phonon drag effect in an ideally pure 2D system under a quantizing magnetic field. Two situations are considered, i.e. when phonons are (i) those of a diffusive heat flow and (ii) those of a focused ballistic beam. In the first case, we calculate the drag current for different asymptotical temperature intervals and give its relation to the phonon-scattering-induced conductivity. For the 2D gas illuminated by a hot beam of phonons, we analyse the manifestation of their cyclotron absorption both in the drag and in the beam-activated conductivity.

1. Introduction

Over the past few years new methods have been used to study 2D electronic systems in quantizing magnetic fields in addition to the conventional electric transport measurements. For example, acoustic phonons have been applied as a tool to test the local properties of 2D particles. Generally speaking, there are two ways of using the phonons as a tool for 2D electron studies. One of them is similar to that of light absorption. It consists in the registration of events of acoustic wave attenuation or scattering by electrons in fine measurements of heat [1] or surface acoustic wave transport [2,3]. The other is based on the fact [4] that there is a relatively high momentum transfer from the non-equilibrium phonons to the electrons producing a current in the 2D system itself, which in fact is easier to register in the electric measurements [5–7]. In what follows we analyse such a current in the degenerate 2D electron gas dragged by a phonon flow in the bulk semiconductor in a quantizing magnetic field $\boldsymbol{H}$. The present paper deals with the model of ideally pure heterostructures. We avoid any impurity scattering effect, which means that we do not lay claim to an exact description of the localized regime under quantum Hall effect conditions, but rather hope that the following results will describe the characteristic features of the dependence on temperature and H of the effect which result from peculiarities of the 2D electron interaction with phonons in high magnetic fields.

Our idealized system thus includes 2D electrons, phonons and their interaction, the latter being of both deformation and polarization origin in polar materials like GaAs.

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The electrons are free and fill some of the lowest Landau levels (we consider the gas to be degenerate at sufficiently low temperatures). The phonons come to the electron gas from the bulk, their flow being controlled, for example, by the bulk impurity and surface scattering or by umklapp processes. This assumption allows us to look upon the drag as a response to the phonon flow created by some external source with the phonon distribution function determined by the way the flow is excited. To make the analysis more specific, the calculations below will concern the case of a diffusive thermal flow [5] due to a weak lattice temperature gradient in the bulk sample and to the case of a high-temperature beam of focused ballistic phonons created by a hot spot located far from the tested part of the heterostructure [6].

The electron–phonon interaction, in its turn, involves two kinds of processes. One of them is related to the elastic phonon scattering by an electron quantized at the Landau level, while the energy conservation law evidently forbids absorption of low-energy phonons in the intra-Landau-level transition, which makes the situation under consideration very different from the case of 2D metals in zero magnetic field [8,9]. In contrast to pure metals, the drag in a quantized electron gas at low temperatures only appears in the second order of the perturbation theory in the electron–phonon coupling. This makes it problematic to apply directly metallic formulae [10] to the case of quantizing magnetic fields in ideally pure structures and in what follows we develop the formalism for the drag theory relevant to this case. The first section of our communication thus deals with a formal perturbation theory treatment of the phonon-scattering-dragged current in a magnetically quantized 2D electron gas. Sections 3 and partially 6 are devoted to the application of such a perturbation theory to the thermopower drag effect and the drag of electrons by focused beams of ballistic phonons, respectively. In section 4 we derive a relation between the drag and conductivity induced by the phonon scattering. In section 5 we use it when accounting for the effect of dynamical screening of the electron–phonon interaction and then describe the resulting temperature behaviour of the drag thermopower under a quantizing magnetic field, with specific applications to GaAs–AlGaAs hetorostructures.

Concerning the case of ballistic phonon beams, it should be noted that the beam temperature can exceed considerably that of 2D electrons. Thus, in Ssction 6 we also study the manifestation (both in the drag effect and phonon-beam-activated conductivity) of another kind of process: absorption of high-energy phonons which are in resonance with the inter-Landau-level transition.

2. Electron–phonon interaction and drag current at degenerate Landau levels

The problem one encounters in determining the electron current subjected to a quantizing magnetic field arises from the currentless nature of Landau states which is due to the infinite degeneracy of the discrete electronic spectrum. Indeed, the current carried by any state ψ_n is exactly zero in the absence of an external electric field, $i \propto ev = e\,\partial\epsilon/\partial p = 0$. To overcome this difficulty, we use the same trick as is used to describe the transport in the tunnelling Hamiltonian approach [11], when the eigenstates of a non-perturbed Hamiltonian are currentless (localized in one of the electrodes). We define the current $\hat{I}$ as the change in number of particles $\hat{N}$ in a half-space per unit time:

$$\hat{I} = e\,\partial\hat{N}/\partial t = -(\mathrm{i}/\hbar)e[\hat{N},\hat{H}]. \qquad (1)$$

The Hamiltonian $\hat{H}$ in equation (1),

$$\hat{H} = \sum \epsilon_n a^+_{n,p} a_{n,p} + \sum \hbar\omega_q b^+_q b_q + \hat{V} \tag{2}$$

contains three terms: that of only electrons, of free phonons and of their interaction

$$\hat{V} = \sum a^+_{n,p} a_{n',p'} \{V_{np,n'p'}(q) b_q + V^*_{n'p',np}(q) b^+_q\}. \tag{3}$$

In defining the operator for the number of particles in a half-space, we have some freedom due to the localization of states by a magnetic field. To simplify our consideration, we choose the Landau gauge and determine $\hat{N}$ as a number of electrons occupying the states centred in a half-space, for example $y < 0$ [12] (i.e. we define the current component along the y-axis),

$$\hat{N} = \sum w(p) a^+_{n,p} a_{n,p}. \tag{4}$$

Here and in what follows n is the Landau level number, p is the electron momentum along the x-axis (the Landau gauge is used), q is the phonon momentum and $w(p)$ is the step function ($w = 0, p > 0$ and $w = 1, p < 0$). This $\hat{N}$ commutes with the electronic part of the Hamiltonian, and the current operator thus appears via the electron–phonon interaction term in equation (2)

$$\hat{I} = -(\mathrm{i}/\hbar) e \sum a^+_{n,p} a_{n',p'} \{V_{np,n'p'}(q) b_q + V_{np,n'p'}(-q) b^+_q\}(w(p) - w(p')). \tag{5}$$

With the current operator in hand, the calculation of the drag can be performed according to the rules of Keldysh's diagrammatic technique [13]. The diagrams to be accounted for in the low-temperature limit are schematically shown in figure 1. Solid lines correspond to the electron advanced and retarded Green functions ($G^{\mathrm{r,a}}(\epsilon) = [\epsilon - \hbar\omega_{\mathrm{c}} \pm \mathrm{i}\delta]^{-1}$ for the degenerate Landau level) and Keldysh's function G^{K} which is related to the electronic occupation numbers. Dashed lines mean the same for phonons (for example, see [16], problem related to §92). The diagrams contain vertices of two types: the current (I) and the interaction (V) vertices described by equations (5) and (3) respectively. According to the energy conservation rules in the vertices, intermediate electron states that follow the absorption (or emission) of phonons are virtual, which corresponds to $G^{\mathrm{r,a}} = [(n - n')\hbar\omega_{\mathrm{c}} \pm \hbar\omega(q)]^{-1}$. At this point we find that if only soft phonons exist in the system at low temperature, $\hbar\omega(q) \leqslant T \ll \hbar\omega_{\mathrm{c}}$, the inter-Landau-level virtual transitions can be neglected and the Landau level number should be the same along any diagram in figure 1. Keldysh's functions of electrons, $G^{\mathrm{K}} = 2\pi\mathrm{i}(2\nu_n - 1)$, are, therefore, determined by partial filling factors $0 \leqslant \nu_n < 1$, of the occupied Landau levels.

All the diagrams shown in figure 1 correspond to the second-order perturbation theory in the electron–phonon interaction and describe the phonon scattering process only allowed by the energy conservation rules in systems with an infinitely degenerate discrete spectrum. As usual in second-order perturbation theory [14], the amplitude of the two-phonon process is composed of the terms differing by the permutation of phonon absorption and emission vertices. The observables should, therefore, account for both the squares of each of them (diagrams without crossing of phonon lines) and their interference, which explains the presence of the cross diagram in figure 1.

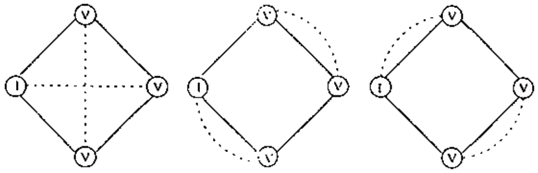

Figure 1. Main perturbation theory diagrams contributing to the phonon-scattering-induced current. We use Keldysh's version of the diagrammatic technique, i.e. solid and dashed lines correspond to advanced, retarded and Keldysh's Green functions [13, 16] of electrons and phonons, respectively.

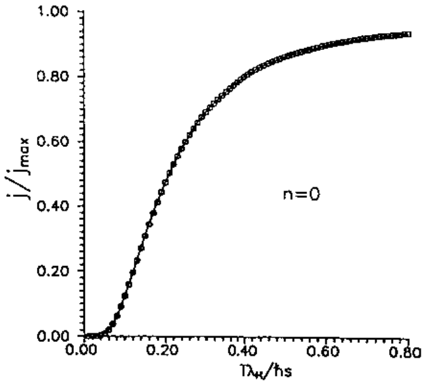

Figure 2. Temperature dependence of the drag current near the saturation regime which shows that the drag current is already saturated ($j_{\rm drag} \approx j_{\rm max}$) at $T\lambda_H/\hbar s \approx \pi^{-1}$. In these calculations we used the lowest Landau level $n = 0$.

Changing the order of absorption and emission vertices, one changes the sign of the Green function of a virtual state hence, diagrams of different topological types have different signs. We thus write an analytical expression for the drag current density $j_{\rm drag}$ (as follows from equation (4)–(5), its projection onto the y-axis) in the form

$$j_{\rm drag} = 2e\nu_n(1-\nu_n)\frac{1}{\hbar^4}\frac{L_x^2L_y^2L_z^2}{(2\pi)^6}\int \mathrm{d}\boldsymbol{q}_1\,\mathrm{d}\boldsymbol{q}_2\frac{\delta(q_1-q_2)}{s^3q_1q_2} \times |V_{n,n}(\boldsymbol{q}_1)V_{n,n}(\boldsymbol{q}_2)|^2(q_{1x}-q_{2x})[f(\boldsymbol{q}_1)-f(\boldsymbol{q}_2)] \times [1-\cos(\lambda_H^2\boldsymbol{l}_z(\boldsymbol{q}_1\times\boldsymbol{q}_2))] \tag{6}$$

where the last multiplier just reflects some partial cancellation between them. In this expression $f(\boldsymbol{q})$ is the phonon distribution function and $V_n(\boldsymbol{q})$ the electron–phonon interaction matrix element dependent on the electron–phonon coupling mechanism in the material. In what follows we restrict ourselves to the case of cubic polar crystals (for example, GaAs) with the deformation potential produced only by longitudinal phonons. So long as we also ignore the difference between the longitudinal and the transverse sound velocity (s), the matrix element can be written as

$$|V_{n,n}(\boldsymbol{q})|^2 = \left[\left(q^2\Xi^2+\beta^2\right)\hbar/(2\mu_0 sqL_xL_yL_z)\right]\mathrm{e}^{-(q_\parallel\lambda_H)^2/2}\left[L_n^0(q_\parallel^2\lambda_H^2/2)\right]^2.$$

In this equation we follow the notation in [15]: Ξ is the deformation potential interaction constant, β the polarization interaction constant summed over transverse and longitudinal phonon modes and $\mu_0 = M_0/a_0^3$ the mass per unit crystalline cell. Further, $q_\parallel$ is the phonon momentum projection to the plane of 2D gas and λ_H the magnetic length. The construction containing the normalized generalized Laguerre polynomials L_n^0 results from the specific form of the Landau wave functions; as well as considering electrons as purely two-dimensional particles, we neglect extension of their wave functions across the 2D layer (in the direction $\boldsymbol{l}_z$).

3. Current dragged by diffusive phonon flow

Calculating the current defined by equation (6) under specific conditions one needs a specific form of the phonon distribution function. In the case of diffusive heat flow it is formed by the impurity and boundary scattering or umklapp processes and can be written as [16]

$$f = f_T + \delta f \qquad \delta f(\boldsymbol{q}) = -\boldsymbol{q}\cdot \boldsymbol{V}\,(\partial f_T(\omega)/\partial\omega).$$

In this equation $\boldsymbol{V} \propto \nabla T$ is the phonon gas velocity with respect to the lattice and f_T is the equilibrium Bose distribution function. At low temperatures the surface scattering dominates and, therefore [16], $\boldsymbol{V} \propto T^{-1}\nabla T$. After relevant substitutions the drag current in equation (6) takes the form

$$\begin{aligned} j_{\mathrm{drag}} = {} & e\nu_n(1-\nu_n)\frac{[\boldsymbol{l}_z \times \boldsymbol{V}]}{(2\pi)^6\hbar^4}\int \frac{\mathrm{d}q_1\,\mathrm{d}q_2\,\delta(q_1-q_2)}{s^3q^2} \\ & \times \left(-\frac{\partial f_T(\omega)}{\partial\omega}\right)\left(\frac{\hbar(q^2\Xi^2+e^2\beta^2)}{2\mu_0 sq}\right)^2 (q_{1\|}^2+q_{2\|}^2)[1-J_0(\lambda_H^2 q_{1\|}q_{2\|})] \\ & \times \mathrm{e}^{-(q_{1\|}^2+q_{2\|}^2)\lambda_H^2/2}\left[L_n\left(\frac{q_{1\|}^2\lambda_H^2}{2}\right)L_n\left(\frac{q_{2\|}^2\lambda_H^2}{2}\right)\right]^2 \end{aligned} \tag{7}$$

(J_0 is the Bessel function). An example of its temperature dependence for the $n = 0$ Landau level is shown in figure 2, where two different temperature regimes of drag current behaviour are quite distinguishable, depending on the relation between magnetic length and characteristic wavelength, $\hbar s/T$, of a thermal phonon.

At the lowest temperatures, $T < \hbar s/\lambda_H$, when the phonon wavelength is definitely longer than the magnetic length, we are able to replace all the functions in the integral of equation (7) (except $f_T(q)$) by their approximation at $q \to 0$, which simplifies the integration and gives

$$j_{\mathrm{drag}} = e[\boldsymbol{l}_z \times \boldsymbol{V}]\nu_n(1-\nu_n)\frac{32\pi^2}{945}\frac{T^6\lambda_H^4}{(\hbar s)^8}\frac{(e\beta)^4}{(2\mu_0 s^2)^2}F(T/T_*). \tag{8}$$

The factor

$$F(T/T_*) = 1 + \tfrac{8}{5}\sqrt{\tfrac{77}{80}}(T/T_*)^2 + (T/T_*)^4$$

accounts for the interplay between the deformation and polarization contributions to the electron–phonon interaction. At $T < T_*$ the interaction is mostly polar until the crossover temperature

$$T_* = \left(\tfrac{11}{560}\right)^{1/4}\hbar se\beta/\pi\Xi.$$

In GaAs this temperature is about 1 K. At higher temperatures it is the deformation potential interaction that is mainly responsible for the electron–phonon coupling, which enhances the temperature dependence of the current ($j_{\mathrm{drag}} \propto T^9$ for $T > T_*$ competes with $j_{\mathrm{drag}} \propto T^5$ for $T < T_*$).

Another type of asymptotic behaviour of $j_{\rm drag}$ can be expected from *the high-temperature range*, $T \gg \hbar s\sqrt{n}/\lambda_H$ (but still $T \ll \hbar\omega_c$, and the electron gas is thus degenerate), where the high-energy tail of the phonon distribution function is insufficient from the point of view of interaction with quantized electrons. In this case the current is formed by the phonons with wave vectors of the order of $\sqrt{n}/\lambda_H$; hence the dependence of the drag current on the temperature of phonon flow is saturated for any kind of coupling mechanism,

$$j_{\rm drag} = e[\boldsymbol{l}_z \times \boldsymbol{V}]\nu_n(1-\nu_n)\frac{T\lambda_H^{-1}}{(2\pi\hbar s)^3}\frac{(e\beta)^4}{(2\mu_0 s^2)^2}R_n. \tag{9}$$

The factor

$$(R_n \approx 0.05/(n+1/2)\left[1+\theta_n\left(2.5\Xi\sqrt{n+1}/e\beta\lambda_H\right)^2 + \gamma_n\left(2.5\Xi\sqrt{n+1}/e\beta\lambda_H\right)^4\right]$$

accounts for the interplay of the polar and the deformation potential interaction: at $\lambda_H < 2.5\Xi\sqrt{n+1}/e\beta$ the deformation potential interaction is dominant; in GaAs–AlGaAs heterostructures the deformation potential coupling works if the electronic density exceeds $3\times10^{11}\,\rm cm^{-2}$. As mentioned above, $V \sim \nabla T/T$ and for some of the lowest Landau levels $\theta \sim 1$ and $\gamma \sim 0.5$. The current in equation (9) slowly increases with the field and undergoes Shubnikov–de Haas oscillations. Formally, asymptotical behaviour in equation (9) can be expected at $T \gg \hbar s/\lambda_H$, but the comparison of the latter result with the previous one in equation (8) and the numerical calculations of $j_{\rm drag}$ for the lowest Landau level ($n=0$) presented in figure 2 show that such a crossover happens, in fact, at lower temperatures than $\hbar s/\pi\lambda_H$.

4. Relation between drag and phonon-scattering-induced conductivity

In their temperature dependence, the drag coefficients in equations (8)–(9) resemble the phonon-scattering-induced mobility of a quantized electron gas derived in [17] for the electron gas in Si MOSFETs and electrons on the liquid helium surface. This similarity can be followed throughout the above calculations, if we take into account that the external electric field $\boldsymbol{E}$ causes a spatial variation of the electron single-particle energy, and the frequencies of the incident and scattered phonons thus differ by the value of $eE\lambda_H^2\Delta p/\hbar$. One can, therefore, arrive at the expression for the current driven by the external electric field E, just by replacing $[f(q)-f(q')]$ with $eE\lambda_H^2\Delta p\,\partial f_T(\omega)/\partial\hbar\omega$ (or phonon flow velocity $\boldsymbol{V}$ by drift velocity $e[\boldsymbol{l}_z\times\boldsymbol{E}]\lambda_H^2/\hbar$):

$$j_x = \frac{e^2}{\hbar}E\nu_n(1-\nu_n)\frac{\lambda_H^2}{(2\pi)^6\hbar^4}\int \mathrm{d}q_1\,\mathrm{d}q_2\,\frac{\delta(q_1-q_2)}{s^3q^2}\left(-\frac{\partial f_T(\omega)}{\partial\omega}\right)[1-J_0(\lambda_H^2 q_{1\|}q_{2\|})](q_{1\|}^2+q_{2\|}^2)|V_n(q_1)V_n(q_2)|^2. \tag{10}$$

This way of determining the diagonal part of the conductivity gives an idea of some general phenomenological relation between the drag and the phonon-scattering-induced conductivity. Indeed, the temperature gradient in the bulk results in a non-equilibrium phonon distribution function which can be recalculated to the equilibrium one, but in the frame moving with velocity $\boldsymbol{V}$ with respect to the lattice. The electric field $[\boldsymbol{H} \times \boldsymbol{V}]/c$ appearing after the Lorentz transformation results in the current $\sigma_{ij}[\boldsymbol{H} \times \boldsymbol{V}]_j/c$ in the frame moving together with the phonon flow (c is the speed of light). The conductivity tensor σ in this equation is formed just by the scattering between degenerate electrons and equilibrium phonons from the thermal bath. Returning to the basic frame, the dragged current components can be recalculated to give it the form

$$j_{\text{drag}} = \sigma_{xx}[\boldsymbol{H} \times \boldsymbol{V}]/c + \left(\sigma_{xy} - (e^2/h)\nu\right) VH/c \tag{11}$$

which gives the desired relation. The phonon scattering does not change the drift current in crossed electric and magnetic fields, so the classical Hall conductivity relation is still valid and gives exactly zero for the current dragged along the heat flow direction.

In commenting on this general phenomenological result, we should emphasize that it is specific only to the case of ideally pure structures, when both the drag and conductivity have the same origin. In the presence of impurities, the elastic Landau level broadening leads to the possibility of single-phonon absorption events, which motivates the application of a metal-like drag theory to the limit of quantizing magnetic fields, as was developed in [10]. The phenomenological studies of the acousto-electric (drag) effect in the dirty regime [18] show that in the latter case the drag current qualitatively looks like the derivative of the conductivity with respect to density (or filling factor at a constant magnetic field), and its longitudinal component is not exactly zero.

5. Dynamically screened electron–phonon interaction

In deriving equation (11), we made no assumptions on the nature of the electron–phonon coupling or the internal electron gas properties which originated, for example, from the electron–electron interaction. Therefore, we can apply this phenomenological result in extending our consideration to the regime of efficient *dynamical screening of the electron–phonon interaction.* The latter takes place when the local conductivity σ_{xx} is high enough, especially if the speed of charge spreading exceeds the sound velocity, $2\pi\sigma_{xx}/\chi s \gg 1$ [19,20] (χ is the material dielectric constant).

In our estimation we view the dynamical screening of a single-phonon potential as electron redistribution caused by their multiple scattering with the other phonons from the bath. Following the method proposed in [20], we describe such a charge redistribution in terms of a local conductivity and, therefore, renormalize the electron–phonon coupling V^2 by using the dynamical screening factor

$$|V_{n,n}|^2 \to |V_{n,n}|^2/(1 + (2\pi\sigma_{xx}/\chi s)^2).$$

After this procedure, equation (10) acquires the meaning of a non-linear self-consistent equation for the conductivity σ_{xx}. In solving it, we find

$$\sigma_{xx} = 1.32\frac{s\chi}{2\pi}\left(\frac{T\lambda_H}{\hbar s}\right)^{6/5}[\nu_n(1-\nu_n)F(T/T_*)]^{1/5}\left(\frac{e^2}{s\chi\hbar}\right)^{1/5}\left(\frac{(e\beta)^2}{2\mu_0 s^3\hbar}\right)^{2/5} \tag{12}$$

and

$$\sigma_{xx} = B_n(s\chi/2\pi)\left(T^2\Xi^2/2\mu_0 s^5\hbar^3\right)^{2/5}(e^2/s\chi\hbar)^{1/5}[\nu_n(1-\nu_n)]^{1/5} \tag{13}$$

for $T < \hbar s/\lambda_H$ and $T > \hbar s/\lambda_H$, respectively. The numerical coefficients in equation (13) are $B_0 = 0.59$ and $B_{n\gg 1} \approx 0.62$; in GaAs $e^2/s\chi\hbar \sim 30$.

Equations (12)–(13) give some phonon contribution to the local conductivity. We do not claim that these equations can describe the conductivity behaviour even in pure high-mobility structures at high temperatures because under quantizing fields the conductivity extracted from the conventional transport measurements is not a local bulk-averaged property but is formed by the percolation structure and inhomogeneity of the sample. In contrast, the drag effect is originally local and its derivation via equation (11) is sensible, but care should be taken to ensure appropriate recalculation of this local quantity to give the thermopower observed. The corresponding drag current behaviour in these two limits immediately results from the above phenomenological relation and shows rather a weak temperature dependence at the highest temperatures:

$$j_{\text{drag}} \propto [\nu_n(1-\nu_n)]^{1/5}\begin{cases}[TH^2F(T/T_*)]^{1/5} & T < \hbar s/\lambda_H\\ T^{-1/5}H^{-1} & T > \hbar s/\lambda_H\end{cases}. \tag{14}$$

Finally, we can distinguish the following regimes of the drag effect depending on the phonon bath temperature (in arranging them we refer to GaAs, as the most frequently used material and assume half-integer filling at magnetic fields in the range $H \sim 5\,\text{T}$). At the lowest temperatures, $T < T_* = (11/560)^{1/4}\hbar se\beta/\pi\Xi$, the drag is formed by the polar coupling between electrons and phonons and depends as $j_{\text{drag}} \propto T^5\nabla T$ on the temperature of phonon flow. At $T > T_*$ (in GaAs $T_* \sim 1\,\text{K}$) the deformation potential interaction starts dominating, which strongly increases temperature dependence: in this limit $j_{\text{drag}} \propto T^9\nabla T$. At $T \sim \hbar s/\pi\lambda_H$ the drag current takes its maximal value, as shown in figure 2. Then, as temperature increases sufficiently to allow the (local) conductivity induced by the phonon scattering to be comparable with the sound velocity the phonon interaction with electrons is reduced by the dynamical screening. For this limit equation (14) gives $j_{\text{drag}} \propto T^{-1/5}\nabla T$, so the drag starts decreasing at this temperature. In all the regimes, the current is dragged perpendicularly to the phonon flow and its longitudinal component is exactly zero.

Although the detailed comparison of our theory with the experimental results [5] is impeded by the difficulty mentioned one paragraph above, the qualitative behaviour observed is similar to our expectations. When discussing the lowest Landau level, the measured current was increasing rapidly, with saturation at $T \sim \hbar s/\pi\lambda_H$. At higher T a noticeable decrease of thermoelectricity was found in the measurement on a half-filled Landau level (the conductivity in equation (13) is maximal when $\nu = n + \frac{1}{2}$). We can assign this to the dynamical screening effect which should be already developed (in GaAs–AlGaAs heterostructures) at $T \sim 10\,\text{K}$. The values of the thermoelectric coefficient which we calculated near the maximum at $T \sim \hbar s/\pi\lambda_H$ have, surprisingly, the same order of magnitude as the observed ones, though we use no fitting parameters. (These were obtained by substituting the thermal flow velocity V extracted according to the usual rules of [16] from the thermal conductivity value (the latter was measured in the same experiment [5]) into equation (9).) On the other hand, the impurity scattering broadens the Landau levels and allows the first-order phonon absorption channel. The success of an alternative description of the same experimental data to that proposed in [10] leaves open the question of when each of these two theories can be applied. In this connection, we should also mention that our treatment cannot explain the difference found between different samples, where the drag current in one may be several times that found in another.

6. 2D gas illumination by a focused beam of ballistic phonons

Another experimental situation was recently realized [6] in the studies of a 2D gas in GaAs–AlGaAs heterostructures using beams of ballistic phonons emitted from a hot spot in the semiconductor bulk and focused (due to the sound velocity anisotropy) along one of the chosen crystallographic directions $\boldsymbol{l}_{\text{beam}}$ of the lattice. It is natural to parametrize the beam by its temperature T_{beam}, assuming the distribution function to have the form

$$f(\boldsymbol{q}) = f_T(q) + \delta(\boldsymbol{q}/q - \boldsymbol{l}_{\text{beam}}) f_{T_{\text{beam}}}(q).$$

The beam temperature is usually high enough and we shall, therefore, be concerned with the limit of $T_{\text{beam}} \gg \hbar s\sqrt{n}/\lambda_H$. On substituting this distribution function into equation (6) and integrating over phonon wave vector orientations, we get the phonon-scattering drag current

$$\boldsymbol{j}_{\text{drag}} \propto e[\boldsymbol{l}_z \times \boldsymbol{l}_{\text{beam}}]\nu_n(1-\nu_n)\{T_{\text{beam}}\lambda_H^{-5}/(\hbar s)^2\}\,\Xi^4/(2\mu_0 s^2)^2 \tag{15}$$

which shows usual Shubnikov–de Haas oscillations.

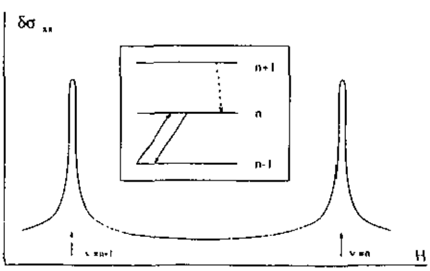

Figure 3. Schematic form of the Shubnikov–de Haas oscillations of conductivity change $\delta\sigma_{xx}$ activated by the phonon beam irradiation. The inset shows inter-Landau-level phonon-assisted transitions accounted for in equation (16).

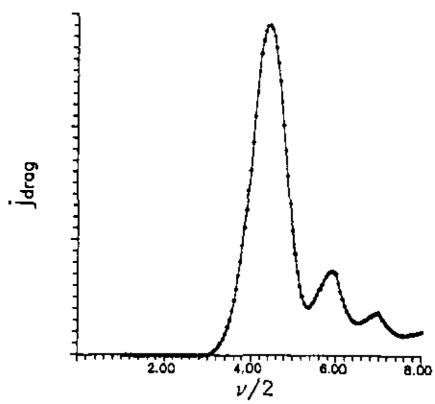

Figure 4. Characteristic form of Shubnikov–de Haas oscillations of the inter-Landau-level part of the drag current activated by a focused beam of ballistic phonons. j_{drag} was calculated from equations (17) for GaAs–AlGaAs heterostructure with electron density $3 \times 10^{11}\ \text{cm}^{-2}$ and for beam inclination 45°.

As distinct from the thermoelectric drag effect, the application of focused ballistic phonon beams allows us to subject the electron gas to high-energy phonon illumination and investigate some kind of resonant cyclotron phonon absorption in the system. The probability of such an event is rather low because of the high momentum of an absorbed phonon, $q_{\|} \sim \omega_c/s \gg \lambda_H^{-1}$. The resulting overheating of the electron gas is weak, and the latter thus retains degeneracy in spite of a high beam temperature. The kinetic equation which describes phonon-assisted electron transitions between some Landau levels close to the Fermi energy (they are shown in the inset to figure 3) can be written as

$$[\nu_k - \nu_{k+1}] f_{T_{\text{beam}}}\left(\frac{\omega_c}{s}\right) \frac{\omega_c^2}{s^2} \left| V_{k,k+1}\left(\boldsymbol{l}_{\text{beam}} \frac{\omega_c}{s}\right) \right|^2 = \nu_{k+1}(1 - \nu_k) \int \mathrm{d}\boldsymbol{q}\, |V_{k+1,k}(\boldsymbol{q})|^2 \delta(q - \omega_c/s). \tag{16}$$

Here $k = n, n-1$. The left-hand side of this equation represents absorption and stimulated emission of phonons with $\boldsymbol{q} = \boldsymbol{l}_{\text{beam}}\omega_c/s$. The right-hand side describes spontaneous emission of phonons with arbitrarily oriented wave vectors. In the above limit of $\omega_c/s \gg \lambda_H^{-1}$ the inter-Landau-level matrix element $V_{k,k+1}(\boldsymbol{l}_{\text{beam}}\omega_c/s)$ is exponentially small for all reasonable angles of beam inclination, and spontaneously emitted phonons thus have momenta almost perpendicular to the gas plane. This allows us to perform the integration in the right-hand side of equation (16) in a general form and then find that

$$\nu_{k+1}(1 - \nu_k) = [\nu_k - \nu_{k+1}](T_{\text{beam}}/ms^2)\,\left[L_k^1(\eta)\right]^2 \eta \mathrm{e}^{-\eta}$$

where $\eta = \omega_c^2 \lambda_H^2 (\boldsymbol{l}_z \times \boldsymbol{l}_{\text{beam}})^2 / 2s^2 = (\hbar\omega_c/2ms^2)(\boldsymbol{l}_z \times \boldsymbol{l}_{\text{beam}})^2$. This equation immediately shows that the conductivity increase under illumination conditions due to the non-equilibrium repopulation of Landau levels is most prominent near the integer filling factors, where more non-equilibrium electrons can be accumulated at a higher Landau level,

$$\delta\sigma_{xx} \propto \delta\nu_{n+1} = \nu_{n+1} - \nu_{n+1}^0 = \sqrt{T_{\text{beam}}/ms^2}\, L_n^1(\eta)\eta^{1/2}\mathrm{e}^{-\eta/2}.$$

At a partially filled Landau level $\delta\sigma_{xx} \sim \delta\nu_k \sim (T_{\text{beam}}/ms^2)(L_n^1(\eta))^2\eta\mathrm{e}^{-\eta}$ is much smaller. We expect, therefore, a splash-like shape of the Shubnikov–de Haas oscillations of phonon-beam-activated conductivity: in a narrow region near integer filling, the exponential factor changes from $\exp(-\eta/2)$ to $\exp(-\eta)$, which can compete with the fast change of the exponent itself due to variation of a magnetic field. Figure 3 schematically shows a qualitative behaviour of this effect. We should also mention that the intra-Landau-level scattering by means of low-frequency phonons leads to the contribution to $\delta\sigma_{xx}$ which has minima at integer filling, and is thus distinguishable from the resonance effect. Hence, in extracting the activated change of conductivity from the above behaviour, one can register the events of the cyclotron absorption of acoustic phonons in the ballistic beam experiments at rather low magnetic fields.

As for the resonant drag current, it appears due to an imbalance between absorption and stimulated emission processes and can be calculated as

$$j_{\text{drag}} = [\boldsymbol{l}_z \times \boldsymbol{l}_{\text{beam}}](2\pi e/\hbar)\{\omega_c^2\left[\omega_c^2 \Xi^2 + (e\beta s)^2\right]/2\mu_0 s^6\} \times [\nu_{N+1}(1 - \nu_N) + \nu_N(1 - \nu_{N-1})] \tag{17}$$

which gives the value (for $\hbar\omega_c > \hbar s e\beta/\Xi \sim 1\,\text{meV}$ in GaAs)

$$j_{\text{drag}} = \frac{2\pi e T_{\text{beam}}}{\hbar s} \frac{\Xi^2(2ms/\hbar)^3}{2\mu_0 s^2 \hbar} \frac{\eta^5 e^{-\eta}}{|l_z \times l_{\text{beam}}|^7} \sum_{k=n-1,n} (\nu_k^0 - \nu_{k+1}^0)[L_k^1(\eta)]^2.$$

This equation shows that the resonant drag current is exponentially small at high fields and the exponential magnetic field dependence practically obliterates the Shubnikov–de Haas oscillations in the extreme quantum limit. We illustrate this in figure 4 for some of the lowest Landau levels and in such cases the drag occurs via second-order processes; at a lower magnetic field (or at higher filling) the resonant drag current suddenly increases and begins to dominate.

Finally, under the conditions of the quantum Hall effect, $\nu = n$, the non-equilibrium stationary state found from equation (16) contains a number of electron–hole pairs (at the nth Landau level) with electron oscillator centre shifted with respect to its abandoned empty place. This produces some electric diople polarization of a system. When the sample has the form of a strip, we expect certain charge accumulation at the edges, resulting in an electric field across the sample. The latter should give rise to the excess (dissipationless) Hall current in the system.

7. Conclusions

In summary, we have studied the phonon drag of 2D electrons under a quantizing magnetic field and have derived some general relations between the drag and conductivity in the model of free 2D electrons interacting with bulk phonons. The characteristic temperatures of a crossover between different asymptotical regimes resulting from the features of coupling of bulk phonons with 2D electrons at Landau levels have been also determined. One of them, T_*, separates regions where either the polar or deformation potential mechanism of coupling dominates. The second, $T \sim \hbar s\sqrt{n}/\pi\lambda_H$, is related to the specific structure of the Landau functions and manifests some cut-off of the short-wavelength phonons from the current formation. Finally, the third points to such an increase of local phonon-scattering-induced conductivity which provides efficient enough dynamical screening of the electron–phonon interaction itself. The latter two effects are the source of the saturation and even decrease of the drag current with increasing temperature.

As far as the case of focused ballistic phonon beams is concerned, we believe that here the resonant cyclotron phonon absorption is the most interesting subject for discussion. Our results show that the beam-activated conductivity possesses different Shubnikov–de Haas oscillation shapes when it originates from phonon scattering or resonant absorption events, which makes them distinguishable. On the other hand, we claim that in an extreme quantum limit the drag by focused ballistic beams mainly results from the intra-Landau-level scattering processes. We also predict an excess Hall current in the system excited by partial polarization of electronic gas under the phonon beam illumination.

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References

[1] Eisenstein I P, Gossard A C and Narayamurti V 1987 *Phys. Rev. Lett.* **59** 1341
[2] Wixforth A, Kotthaus J P and Weimann G 1986 *Phys. Rev. Lett.* **56** 2104
Wixforth A, Scriba J, Wassermeier M, Kotthaus J P, Weimann G and Schlapp W 1989 *Phys. Rev.* B **40** 7874
[3] Iordanskii S V and Musikantskii B A 1989 *Sov. Phys.–JETP* **69** 1006
[4] Parmenter R H 1953 *Phys. Rev.* **89** 990
Weinreich G and White H G 1957 *Phys. Rev.* **106** 1104
[5] Fletcher R, Maan J C and Weimann G 1985 *Phys. Rev.* B **32** 8477; 1986 *Phys. Rev.* B **33** 7122; 1988 *J. Phys. C: Solid State Phys.* **21** 2681
[6] Karl H, Dietsche W, Fischer A and Ploog K 1988 *Phys. Rev. Lett.* **61** 2360
[7] Esslinger A, Wixforth A, Winkler R W and J P Kotthaus 1992 *unpublished*
[8] Ruf C, Obloch H, Junge B, Gmelin E, Ploog K and Weimann G 1988 *Phys. Rev.* B **37** 6377
Oxley J P, Gallagher B, Galloway T, Butcher P, Fromhold T, Mulimani B and Karavolas V 1991 *Proc. 20th Int. Conf. on Semiconductor Physics (Thessaloniki, 1990)* ed E Anastassakis and J Joannopoulos (Singapore: World Scientific) p 853
[9] Cantrell D G and Butcher P N 1985 *J. Phys. C: Solid State Phys.* **20** 1993
Smith M J and Butcher P N 1989 *J. Phys.: Condens. Matter* **1** 1261
[10] Kubakaddi S S, Butcher P N and Mulimani B G 1989 *Phys. Rev.* B **40** 1377
Lyo S K 1989 *Phys. Rev.* B **40** 6458
[11] Duke C B 1969 *Tunneling in Solids (Solid State Physics)* ed H Ehrenreich, F Seitz and D Turnbull (New York: Academic)
[12] The degeneracy of the Landau levels allows different definitions of $\hat{N}$. The more usual form of $\int_{y<0} \hat{\psi}^{+}(y)\hat{\psi}(y)\,dy$ gives the same results, but needs more unwieldy calculations.
[13] Keldysh L V 1965 *Sov. Phys.–JETP* **20** 1018
[14] Holstein T 1959 *Phys. Rev.* **119** 479
[15] Gantmakher V F and Levinson I B 1987 *Carrier Scattering in Metals and Semiconductors (Modern Problems in Condensed Matter Science* (Amsterdam: North-Holland) ch 4
[16] Lifshitz E M and Pitaevskii L P 1981 *Physical Kinetics* (Oxford: Pergamon) p 297
[17] Bychkov Yu A, Iordanskii S V and Eliashberg G M 1981 *JETP Lett.* **34** 473
Iordanskii S V and Meshkov S V 1991 *J. Low Temp. Phys.* **83** 273
[18] Fal'ko V, Meshkov S and Iordanskii S V, unpublished
[19] Govorov A O and Chaplik A V 1989 *Sov. Phys.–JETP* **68**, 1143
[20] Efros A L and Galperin Yu M 1990 *Phys. Rev. Lett.* **64** 1959